1. $\frac{d^2x}{dy^2}$ equals

(A) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

2. Let f(x) and g(x) are differentiable functions such that $\frac{f(x)}{g(x)} = 7$. If $\frac{f'(x)}{g'(x)} = \alpha$ and $\left(\frac{f(x)}{g(x)}\right)' = \beta$ (f′(x) represents the derivative of f(x) w.r.t. x), then $\frac{\alpha-\beta}{\alpha+\beta} =$ _____

(A) 0 (B) 1 (C) 7 (D) None of these

3. Let f(x) = $(x^2 - 3x + 2)\ |x^3 - 6x^2 + 11x - 6| + \left|\sin\left(x+\frac{\pi}{4}\right)\right|$. Number of points at which the function f(x) is non-differentiable in [0, 2π], is

(A) 5 (B) 4 (C) 3 (D) 2

4. If f(x) = $\begin{cases} x\sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$, then

(A) Both $f'(0^+)$ and $f'(0^-)$ Do not Exist (B) $f'(0^+)$ exist but $f'(0^-)$ does not

(C) $f'(0^+) = f'(0^-)$ (D) None of these

5. If $f(x) = \cos x - \int_0^x (x-t)f(t)dt$, then $f''(x) + f(x)$ equals

(A) $-\cos x$ (B) 0 (C) $\int_0^x (x-t)f(t)dt$ (D) $-\int_0^{-x}(x-t)f(t)dt$

6. If f (x) is differentiable and $\int_0^{t^2} x f(x)dx = \frac{2}{5}t^5$, then $f\left(\frac{4}{25}\right)$ equals

(A) 2/5 (B) - 5/2 (C) 1 (D) 5/2

7. If $f(x) = \int_{-1}^{1}\frac{\sin x}{1+t^2}dt$, then $f'\left(\frac{\pi}{3}\right)$ is

(A) Non - existent (B) $\frac{\pi}{4}$ (C) $\frac{\pi\sqrt{3}}{4}$ (D) none of these

8. If $f(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2f'(t))dt$ then $f'(4)$ is equal to

(A) 16 (B) $\frac{32}{9}$ (C) $\frac{32}{3}$ (D) none of these

9. Let f(x) be a differentiable function such that $f'(x)+f(x)=4xe^{-x}\cdot\sin 2x$ and f(0) = 0. Then the value of $\lim_{n\to\infty}\sum_{k=1}^{n}f(k\pi)$ is/are

(A) $-\frac{2\pi e^{\pi}}{\left(e^{\pi}-1\right)^2}$ (B) $\frac{2\pi e^{\pi}}{\left(e^{\pi}-1\right)^2}$ (C) $-\frac{2\pi e^{\pi}}{\left(e^{\pi}+1\right)^2}$ (D) $\frac{2\pi e^{\pi}}{\left(e^{\pi}+1\right)^2}$

10. If x = f(t) and y = g(t) are differentiable functions of t then $\frac{d^2y}{dx^2}$ is

(A) $\frac{f'(t)g''(t)-g'(t)f''(t)}{\left(f'(t)\right)^3}$ (B) $\frac{f'(t)g''(t)-g'(t)f''(t)}{\left(f'(t)\right)^2}$

(C) $\frac{g'(t)f''(t)-f'(t)g''(t)}{\left(f'(t)\right)^3}$ (D) $\frac{g'(t)f''(t)+f'(t)g''(t)}{\left(f'(t)\right)^3}$

11. Let f: R→R be a continuous & differentiable function given by $f(x)=x+\int_0^1(xy+x^2)f(y)dy$. Then

(A) $\int_0^1 f(x)dx=\frac{26}{23}$ (B) $\int_0^1 f(x)dx=\frac{25}{13}$ (C) $\int_0^1 xf(x)dx=\frac{13}{25}$ (D) $\int_0^1 xf(x)dx=\frac{25}{23}$

12. If f(x) = $\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}$, $g(x)=\int_0^{\cos x}\left(1+\sin t^2\right)dt$ then the value of $f'\left(\frac{\pi}{2}\right)=$

(A) 1 (B) - 1 (C) 0 (D) ½

13. If g is the inverse of f & $f'(x)=\frac{1}{1+x^5}$ then $g'(x)$ equals ___

(A) 1 + $[g(x)]^5$ (B) $\frac{1}{1+\left[g(x)\right]^5}$ (C) - $\frac{1}{1+\left[g(x)\right]^5}$ (D) None of these

14. If $y=f\left(\frac{3x+4}{5x+6}\right)$ & $f'(x)=\tan\left(x^2\right)$ then $\frac{dy}{dx}=$

(A) $\tan(x^3)$ (B) $-2\tan\left[\frac{3x+4}{5x+6}\right]^2\times\frac{1}{(5x+6)^2}$

(C) $f\left(\frac{3\tan\left(x^2\right)+4}{5\tan\left(x^2\right)+6}\right)\cdot\tan\left(x^2\right)$ (D) None of these

15. Let f(x) = $\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is a constant. Then $\left.\frac{d^3\left[f(x)\right]}{dx^3}\right|_{x=0}$ is:

(A) $6p^3$ (B) $p + p^2$ (C) $p + p^3$ (D) independent of p

16. If f(x) = $(2x - 3\pi)^5$ + $\frac{4x}{3}+\cos x$ and g is the inverse function of f, then $g'(2\pi)$ is equal to:

(A) 7/3 (B) 3/7 (C) $\frac{30\pi^4+4}{3}$ (D) $\frac{3}{30\pi^4+4}$

17. If $y=\frac{\cos 6x+6\cos 4x+15\cos 2x+10}{\cos 5x+5\cos 3x+10\cos x}$, then $\frac{dy}{dx}=$.......

(A) 2 sin x + cos x (B) - 2 sin x (C) cos 2x (D) sin 2x

18. $\lim_{x\to 0^+}\left(x^{x^x}-x^x\right)$ is equal to:

(A) 0 (B) 1 (C) - 1 (D) D.N.E.

19. If $x_1, x_1, x_2, x_3, \ldots, x_{n-1}$ be n zero's of the polynomial $P(x) = x^n + \alpha x + \beta$, where $x_i \neq x_j\ \forall\ i, j \in \{1, 2, 3, \ldots, n-1\}$. The value of $Q(x) = (x_1 - x_2)(x_1 - x_3)(x_1 - x_4)\ldots\ldots(x_1 - x_{n-1})$, is:

(A) $n(n-1)x_1^{n-2}$ (B) ${}^nC_2 x_1^{n-2}$ (C) $n x_1^{n-1} + \alpha$ (D) Zero

20. If $\sin x=\frac{2t}{1+t^2}$ & $\cot y=\frac{1-t^2}{2t}$, then the value of $\frac{d^2x}{d^2y}$, is equal to:

(A) 0 (B) 1 (C) - 1 (D) 1/2

21. The value of $\lim_{x\to 0^+}\left(x^x+(\tan x)^{\operatorname{cosec} x}+(\operatorname{cosec} x)^{\tan x}\right)$ is equal to:

(A) 1 (B) 2 (C) $2+\frac{1}{e}$ (D) $1+\frac{1}{e}$

22. If a differentiable function $f(x) = e^x + 2x$ is given, then $\frac{d}{dx}\left(f^{-1}(x)\right)$ at $x=f(\ln 3)$ is equal to:

(A) 1/5 (B) 3/7 (C) 7/3 (D) 5

23. For the curve $32x^3y^2 = (x + y)^5$, the value of $\frac{d^2y}{dx^2}$ at P(1, 1) is equal to:

(A) 0 (B) 1 (C) - 1 (D) 1/2

24. Let f: (- 2, 2) → R be a differentiable function such that f(0) = - 1 and $f'(0)=1$. If $g(x)=\left(f(2f(x)+2)\right)^2$ then $g'(0)$ is equal to:

(A) - 4 (B) 0 (C) - 2 (D) 4

25. Let $f(x)=\log_3\left(\frac{1-x}{1+x}\right)+\log_3\left(x+\sqrt{x^2+1}\right)$ then:

(A) The graph, y = f(x) symmetric about y – axis (B) f(0) = 1

(C) $f'(0)=0$ (D) $f''(0)=0$

26. Let A, B, P be the points the curve y = lnx with their x co-ordinate as 1, 2 and t respectively then the value of $\lim_{t\to\infty}\cos\angle BAP$ is:

(A) $\sqrt{1+\ln^2 2}$ (B) ln 2 (C) $\frac{1}{\sqrt{1+\ln^2 2}}$ (D) $\frac{1}{1+\ln 2}$

27. If $8f(x)+6f\left(\frac{1}{x}\right)=x+5$ and $y = x^2 f(x)$, then $\left.\frac{dy}{dx}\right|_{x=-1}$ is equal to:

(A) 0 (B) $\frac{1}{14}$ (C) $-\frac{1}{14}$ (D) none of these

28. If $x^p x^q=(x+y)^{p+q}$ then $\frac{dy}{dx}$ is

(A) independent of p (B) independent of q

(C) dependent on both p & q (D) $\frac{y}{x}$

29. Let f: R→R, g: R→R and h: R→R be differentiable function such that $f(x) = x^3 + 2x + 1$, $g(f(x)) = x$ and $h(g(g(x))) = x \ \forall x \in R$. Then

(A) $g'(1) = 1/2$ (B) $h'(0) = 10$

(C) If $x_0 \in R$, $x_0^3 + 2x_0 - 2 = 0$ then $h(x_0) = 34$ (D) $h(g(2)) = 12$

30. Let a, b ∈ R and f: R→ R be defined by $f(x) = a\cos\left(\left|x^3 - x\right|\right) + b|x|\sin\left(\left|x^3 + x\right|\right)$. Then f is__

(A) Differentiable at x = 0 if a = 0 and b = 1 (B) Differentiable at x = 1 if a = 1 and b = 0

(C) **NOT** differentiable at x = 0 if a = 1 and b = 0 (D) **NOT** differentiable at x = 1 if a = 1 and b = 1

31. If $\Delta(x) = \begin{vmatrix} x^2 + 4x - 3 & 2x + 4 & 13 \\ 2x^2 + 5x - 9 & 4x + 5 & 26 \\ 8x^2 - 6x + 1 & 16x - 6 & 104 \end{vmatrix} = ax^3 + bx^2 + cx + d$, then

(A) a = 3 (B) b = 0 (C) c = 0 (D) none of these

32. Let $f:\left[-\frac{1}{2}, 2\right] \to R$ & $g:\left[-\frac{1}{2}, 2\right] \to R$ be function defined by $f(x) = [x^2 - 3]$ & $g(x) = |x|f(x) + |4x - 7|f(x)$, where [y] denotes the greatest integer less than or equal to y for y∈R. then

(A) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$

(B) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$

(C) g is NOT differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$

(D) g is NOT differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

33. Given $\dfrac{\int_{f(y)}^{f(x)} e^t dt}{\int_y^x \frac{1}{t} dt} = 1 \ \forall x, y \in \left(\frac{1}{e^2}, \infty\right)$ Where $f(x)$ is continuous & differentiable function s.t. $f\left(\frac{1}{e}\right) = 0$.

If $g(x) = \begin{cases} e^x, & x \geq k \\ e^{x^2}, & 0 < x < k \end{cases}$, then

(A) $f(g(x))$ is continuous for $k = 1$ (B) $f(g(x))$ is differentiable for $k = 1$

(C) $f(g(x))$ is non-differentiable for $k = 1$ (D) $f(g(x))$ is continuous for $k = 2$

34. The function $f(x) = \begin{cases} |x - 3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ is

(A) Continuous at x = 1 (B) differentiable at x = 1

(C) Continuous at x = 3 (D) differentiable at x = 3

35. Let 'f' be a differentiable function satisfying $f(x + y) = f(x) + f(y) + (e^x - 1)(e^y - 1) \ \forall x, y \in R$ and $f'(0) = 2$. Identify the correct statement(s):

(A) $\lim\limits_{x\to 0}\dfrac{f(f(x))}{f(x)-x}=4$ (B) $\lim\limits_{x\to 0}\left(f(x)+\cos x\right)^{1/(e^x-1)}=e^2$

(C) Number of roots of equation f(x) = 0 are 2 (D) Range of f(x) is $(-\infty, \infty)$

36. If $f(x) = x^n$ then find the value of $f(1)+\dfrac{f^1(1)}{1!}+\dfrac{f^2(1)}{2!}+........+\dfrac{f^n(1)}{n!}$ where $f^r(x)$ denotes rth derivative of f(x) w.r.t. x.

37. Let f(x) be a differentiable function such that $f(x) = 1 + \dfrac{x^3}{3}+\int_0^x e^{-t}f(x-t)dt$, if $\int_0^1 f(x)dx=p$ then the value of 5p is _______

38. Let f(x) be a differentiable function in $[-1, \infty)$ and f(0) = 1 such that $\lim\limits_{t\to x+1}\dfrac{t^2f(x+1)-(x+1)^2f(t)}{f(t)-f(x+1)}=1$. Find the value of $\lim\limits_{x\to 1}\dfrac{\ln(f(x))-\ln 2}{x-1}$.

39. If $2x=\left(y^{1/3}+y^{-1/3}\right)$, then find the value of $\dfrac{(x^2-1)}{y}\cdot\dfrac{d^2y}{dx^2}+\dfrac{x}{y}\cdot\dfrac{dy}{dx}$

40. Evaluate: $\lim\limits_{x\to 0}\left(\dfrac{1}{x^5}\int_0^x e^{-t^2}dt-\dfrac{1}{x^4}+\dfrac{1}{3x^2}\right)$

41. If P_n is the sum of GP upto n terms. Show that $(1-r)\dfrac{dP_n}{dr}=nP_{n-1}-(n-1)P_n$.

42. Let $f(x)=x+\cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{2x+....\infty}}}$. Compute the value of $f(5)f'(5)$____

43. Differentiate: $\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$ w.r.t. $\sqrt{1-x^4}$.

44. (a) Let $f(x) = x^2 - 4x - 3$, x > 2 and let g be the inverse of f. Find the value of g′ where f(x) = 2.

(b) Let f, g and h are differentiable functions. If f(0) = 1; g(0) = 2; h(0) = 3; and the derivatives of their pairwise product at x = 0 are (fg)′(0) = 6; (gh)′(0) = 4 and (hf)′(0) = 5 then compute the value of (fgh)′(0).

45. If f : R → R is a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all x ∈ R, then prove that $f(2)=f(1)-f(0)$.

46. If the function $f(x) = x^3 + e^{x/2}$ and $g(x)=f^{-1}(x)$, then the value of $g'(1)$ is____

47. Let $y'(x)+y(x)g'(x)=g(x)g'(x)$, y(0) = 0, x ∈ R, where f′(x) denotes $\dfrac{df(x)}{dx}$ and g(x) is a given non-constant differentiable function on R with g(0) = g(2) = 0. Then the value of y(2) is ____

48. Consider f(x) = (x + 1)(x + 2)(x + 3).......(x + n). Find f′(0).

49. If $y=\dfrac{x^4-x^2+1}{x^2+\sqrt{3}x+1}$ and $\dfrac{dy}{dx}=ax+b$ then find the value of a + b _____

50. If $y=(\ln x)^{(\ln x)^{(\ln x)^{....\infty}}}$. Find $\dfrac{dy}{dx}$

Answer Key

1. D	2. B	3. C	4. A
5. A	6. A	7. B	8. B
9. A	10. A	11. D	12. B
13. A	14. B	15. D	16. B
17. B	18. C	19. B	20. A
21. B	22. A	23. A	24. A
25. D	26. C	27. C	28. D
29. A, B, C	30. A, B	31. B, C	32. B, C
33. (A,B,C)	34. A, B, C	35. A, B, D	36. 2^n
37. 8	38. 1	39. 9	40. 1/ 10
42. 5	43. $\frac{1+\sqrt{1+x^4}}{x^6}$	44. (a) 1/6 (b) 16	46. 2
47. 0	48. $n!\left(1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}\right)$		49. $2 - \sqrt{3}$

50. $\frac{dy}{dx}=\frac{y^2}{x(1-y\ln x)}$